
On a toy network of neurons interacting through nonlinear dendritic compartments

Romain Veltz^{*1}, Nicolas Fournier², and Etienne Tanré¹

¹INRIA – CRISAM - Inria Sophia Antipolis - Méditerranée – France

²Institut de Mathématiques de Jussieu - Paris Rive Gauche – Université Pierre et Marie Curie - Paris 6, Université Paris Diderot - Paris 7, Centre National de la Recherche Scientifique : UMR7586 – France

Abstract

The dendrites of many neurons are endowed with active mechanisms which confer them properties of excitability and enable the genesis of local dendritic spikes. In this work, we consider the propagation of dendritic spikes in a dendrite composed of a single branch. These local dendritic spikes are due to voltage dependent ion channels (i.e. sodium, calcium or NDMA spikes). Because the dendritic compartments are connected with passive conductors, dendritic spikes propagate in both sides, although with possibly different speeds. Two dendritic spikes propagating in opposite directions will cancel out when they collide as in the case of the axon because of the refractory period.

We focus on an abstract description of this nonlinear behaviour which is more amenable to analysis. This description reveals a rich mathematical structure that we study through the use of applied combinatorics. This also provide an algorithm for an efficient simulation. In passing, we link this description to the famous Ulam problem opening the door for a mean field model.

Whenever a dendritic spike reaches the soma, it triggers a depolarization. For simplicity, we put a spiking mechanism in the soma as a generalised integrate and fire model. We call such model, a Ball-and-Stick (BaS) neuron. We then study the large N limit of networks of N excitatory BaS neurons. Among other findings, we are able to extract the right scaling for the synaptic weights which allows to have a large N limit which we derive. Numerical simulations are presented for cases not covered by our mathematical results.

This is one of the first work on mean field limits of networks of spiking neurons with a dendritic branch.

^{*}Speaker