Spectral statistics of sparse random graphs: the Thomae function and modular forms

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Abstract

We discuss the spectral properties of highly sparse adjacency matrices near the percolation threshold. The eigenvalue density of an ensemble of such matrices can be expressed through a discontinuous function at all rational points, known as the "Thomae" (or "popcorn") function, obtained via the construction known as the "Euclid’s orchard." We discuss the connection between the Thomae function and the theory of modular forms and propose a continuous approximation of the Thomae function on the basis of the Dedekind’s eta-function near the real axis. We propose simple arguments that demonstrate the presence of the "Lifshitz tail" near the spectral boundary typical for the one-dimensional Anderson localization.